## AKADEMEIA

## HIGH SCHOOL

## academic potential test for future A-level students 2020

## Time allowed: 90 minutes

Instructions:

- Use black ink or black ball-point pen.
- Answer all questions.
- You are allowed to use a calculator.
- Unless otherwise stated in a question, all numerical answers must be given either exactly or correct to 3 significant figures.

Information:

- The test consists of a few sections.
- Note that the last section is for students who consider doing Further Mathematics in A-levels.

| Questions | Mark |
| :---: | :---: |
| $1-4$ | $/ 17$ |
| $5-7$ | $/ 10$ |
| $8-10$ | $/ 7$ |
| $11-14$ | $/ 13$ |
| $15-17$ | $/ 18$ |
| $\left({ }^{*}\right) 18-24$ $/ 55$ <br> total  <br> Standard  <br> total  <br> Advanced  <br> SL score  <br> AL score  |  |

## ARITHMETIC \& ALGEBRA

Q1. [8 marks]
Express in terms of $a$. Give your answer in exact form.
Q1.1 $\sqrt[3]{a^{-2} a^{8}}$
$\qquad$
Q1.2 $\sqrt{a^{6} \times\left(1 \frac{1}{3}\right)^{-2}}$

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Q1.3 $\left(\frac{1}{4} a^{-2}\right)^{3} \div\left(\frac{1}{2} a^{-3}\right)^{4}$

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Q1.4 $\frac{\left.(2 a)^{8} 4 a^{2}\right)^{7}}{\left(16 a^{3}\right)^{8} \div(-8 a)^{3}}$

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Q2. [2 marks]
Find the number whose $22 \%$ is equal 75 . Give your answer to 2 decimal places.

Q3. [3 marks]
Rationalize the denominator of the following fractions. Show your workings fully.
Q3.1 $\frac{\sqrt{8}-4}{\sqrt{2}}$

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Q3.2 $\frac{2 a \sqrt{3}+a}{\sqrt{3}-2}$

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## Q4. [4 marks]

Expand and leave the answer in simplest form.
Q4.1 $\left(4 x+\frac{1}{2}\right)^{2}$

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Q4.2 $(a-b+a b)^{2}$

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Q4.3 $(a \sqrt{2}-3)(a \sqrt{8}+1)-(2 a)(2 a-2 \sqrt{2})$

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## STATISTICS \& PROBABILITY

## Q5. [3 marks]

A fair cubic dice and a fair coin are tossed once. On one side of the coin there is a number 1 and on the other side there is a number 2 . What is the probability that the sum of outcomes is larger than 4 ?

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## Q6. [4 marks]

Consider four different whole numbers that have the following properties:

- their range is 6 ,
- their median is 7 ,
- their mean is 7.5.

Find the numbers.

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Q7. [3 marks]
In a group of 20 students 14 learn French, 9 learn German and 2 do not learn any of the two languages. A student is chosen at random from the group. What is the probability that he learns both French and German?

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## EQUATION OF A LINE

Q8. [2 marks]
Consider the points $(-3,4)$ and $(1,-3)$.
Q8.1 Find the gradient of the line passing through the points. Give your answer as an exact fraction.
$\qquad$
Q8.2 Find the distance between the points.

| $\square$ |
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| $\square$ |

Q9. [3 marks]
Find the equation of a line perpendicular to $y=1.5 x+2$ and passing through point $(6,-1)$. Give your answer in the form $A x+B y+C=0$, where $A, B$ and $C$ are integers.
$\qquad$
Q10. [2 marks]
Find the area of the triangle bounded by the line $y=\frac{2}{3} x-3$ and the coordinate axes.

## EQUATIONS \& INEQUALITIES

Q11. [3 marks]
Find the set of common solutions of the following inequalities.

$$
7-2 x>0 \quad \text { and } \quad 5 x+10 \geq 0 \quad \text { and } \quad|x|<1
$$

$\square$
Q12. [7 marks]
Solve the equations and inequalities. Give all answers in simplest form.
Q12.1 $x^{2}+7 x-8=0$
$\square$
Q12.2 $\frac{x-1}{3}=\frac{2}{x-1}$

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Q12.3 $x+\frac{6}{x}=7$

Q12.4 $x^{2}=5 x$

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Q12.5 $|x+1|=3$

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Q12.6 $x^{2}-2 x<3$

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Q13. [1 marks]
Make $r$ the subject of the formula $F=G \frac{m_{1} m_{2}}{r^{2}}$.

Q14. [2 marks]
Solve the following equations simultaneously.

$$
5 x+4 y=6 \quad \text { and } \quad 3 x-2 y=8
$$

## TRIGONOMETRY

You may like to use the cosine rule in this section: $c^{2}=a^{2}+b^{2}-2 a b \cos C$.
Q15. [3 marks]
In triangle $A B C$ the sides $A B$ and $B C$ are 6 and 4 respectively. The angle at $A$ is $35^{\circ}$. Find the measure of the angle $C$.
$\qquad$
Q16. [2 marks]
Find the measure of the smallest angle in the triangle with sides 3,5 and 7 .

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Q17. [3 marks]

Q17.1 Find the obtuse angle $B$ such that $\sin B=\sin 40^{\circ}$.
$\square$
Q17.2 Find an angle $C$ such that $\cos C=-\sin C$.

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Q17.3 $D$ is an acute angle $\left(0^{\circ}<D<90^{\circ}\right)$. Find the exact value of $\cos D$ if $\sin D=\frac{2}{3}$.
$\square$

For candidates who consider doing A Level Mathematics but not Further Mathematics the exam finishes here. An additional section for students considering Further Mathematics starts on the next page.

## ADVANCED PART

Q18. [4 marks]
Write down an equation of each of the curves shown below.

Q18.1.



Q18.2.



## Q19. [3 marks]

Consider the graph of the function $y=f(x)$ shown below.


In the diagrams on the next page sketch the graphs of the curves with given equations.
Q19.1.


$$
y=\frac{1}{2} f(x)
$$

$$
y=f(2 x)
$$

$$
y=f(x+2)
$$

Q20. [2 marks]
Sketch the graph of

$$
y=x^{2}(x-1)(x-2)
$$

and hence or otherwise solve the inequality

$$
x^{2}(x-1)(x-2)<0 .
$$



Q21. [3 marks]
Which of the numbers is larger?
Q21.1. $\log _{\left(a^{2}\right)}\left(a^{3}\right)$ or $\log _{\left(a^{3}\right)}\left(a^{2}\right)$

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Q21.2. $\left(\frac{1}{2}\right)^{x}$ or $\left(\frac{1}{2}\right)^{x+1}$
$\square$
Q21.3. $\log _{a} b$ or $\log _{(2 a)} b$ when $a>1$

Q22. [1 marks]
Find the distance of the centre of the circle $(x+6)^{2}+(y-8)^{2}=11$ from the origin.
$\qquad$
Q23. [3 marks]
Find the coordinates of the points where the circle $(x-2)^{2}+(y+8)^{2}=100$ intersects the $x$-axis.

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Q24. [2 marks]
What is the constant term of the expansion $\left(x+\frac{2}{x}\right)^{6}$ ?

## DRAFT

